Hence

$$\binom{n}{a}-\binom{i}{1}\binom{n-1}{a}+\binom{i}{2}\binom{n-2}{a}-\cdots+(-1)^i\binom{n-i}{a}=\binom{n-i}{a-i}. \qquad (1)$$

In the case when $i = a$, we have $a - i = 0$, and $\binom{n-i}{a-i} = 1$; therefore, equation (1) becomes

$$\binom{n}{a}-\binom{a}{1}\binom{n-1}{a}+\binom{a}{2}\binom{n-2}{a}-\cdots+(-1)^a\binom{n-a}{a}=1. \qquad (2)$$

When $i > a$, then $a - i$ is negative, so that $\binom{n-i}{a-i} = 0$. Hence for $i > a$,

$$\binom{n}{a}-\binom{i}{1}\binom{n-1}{a}+\binom{i}{2}\binom{n-2}{a}-\cdots+(-1)^i\binom{n-i}{a}=0. \qquad (3)$$

An excellent solution by a somewhat different method from the ones exhibited in these two solutions was received from A. M. HARDING.

425. Proposed by CLIFFORD N. MILLS, Brookings, S. D.

Solve for x and y the equations $2^{x+y} = 6$, $2^{x+1} = 3^y$.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

We shall agree to mean by a^x, where a is real, $e^{x \log a}$, in which the real logarithm of a is to be taken, so that a^x is single valued. We have, then, from the given equations, $(x + y) \log 2 = \log 6$, $(x + 1) \log 2 = y \log 3$, where all the logarithms are real. Solving these linear equations for x and y, we obtain the required solutions

$$x = \frac{(\log 6)^2 - \log 2 \cdot \log 12}{\log 2 \log 6} = 1.198 \text{ and } y = \frac{\log 12}{\log 2} = 1.387.$$

On account of the homogeneity of these fractions in the logarithmic function, we may substitute logarithms to the base 10 in our computation of the numerical values of x and y.

Also solved by EMMA GIBSON, NATHAN ALTSHILLER, A. L. MCCARTY, A. M. HARDING, HORACE OLSON, RICHARD MORRIS, G. W. HARTWELL, C. E. GITHENS, FRANK IRVIN, ELIZABETH B. DAVIS, F. L. CARMICHAEL, EDWARD S. INGHAM, S. A. JOFFE, W. C. EELLS, ELMER SCHUYLER, V. M. SPUNAR, R. M. MATHEWS, and CYRIL A. NELSON.

GEOMETRY.

454. Proposed by LOUIS ROUILLION, Mechanics Institute, New York City.

Show how to construct an equilateral triangle with its vertices lying on three [parallel] lines not equally spaced.

I. SOLUTION BY C. N. SCHMALL, New York City.

The proposer evidently meant that the three lines are parallel.

(i) Let x, y, and z be the three parallel lines in order. At any point, M, on the line y, construct the angles AMC and BMC each equal to 60° (Fig. 1). Draw the line AB, and, about the triangle AMB, circumscribe the circle ABC.